(2) The error is usually about 2 percent or less when the rule is applied to the lower levels of an average curve.

(3) The error is normally of the order of 1 percent at the higher levels (5,000–20,000 feet) of an average curve.

(4) When applied to the upper levels of a very stable curve (e. g. one featuring an extensive inversion), the rule leads to an overestimation of the height which may amount to 4 percent or more in an extreme case. The formula is least accurate when applied to the upper levels of such a curve.

From the fact mentioned above, that in an average situation the percentage error is greatest in the lowest levels, it follows that the absolute error is small at all heights in such a situation, and is usually of the order of 100–200 feet.

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AN EVALUATION OF THE BERGERON-FINDEISEN PRECIPITATION THEORY

By A. R. STICKLEY

[Weather Bureau, Washington, May 1939]

The fundamental concept of the Bergeron-Findeisen precipitation theory was advanced by T. Bergeron (**1**) in 1935. As then formulated, it asserted that, disregarding some rather exceptional cases, the necessary condition for the formation of drops large enough to produce rain of any considerable intensity is that subfreezing temperatures exist in the cloud layer from which the rain descends. Findeisen (**2**) (**3**) has recently amplified this theory by introducing Wegener's postulate as to the existence of two kinds of nuclei—condensation nuclei and sublimination nuclei—on which the water vapor of the earth's atmosphere may respectively condense and sublime. The process thus amplified may be briefly described as follows:

Assuming that the dew-point of a mass of air is higher than the freezing point of water and that the mass of air contains both condensation nuclei (which are generally assumed to be ominipresent) and sublimation nuclei, let it be supposed that it is being cooled by any process or combination of processes. Under these conditions condensation will first take place on the condensation nuclei until the point is reached where the vapor pressure exerted by the sublimation nuclei is less than the vapor pressure exerted by the water droplets—this latter point, as will be shown later, seeming to be, in some cases at least, not far below the temperature of freezing. After this point is reached, any further cooling will cause the water vapor of the atmosphere to sublime on the sublimation nuclei and, at the same time, to be replenished by evaporation from the liquid drops. These latter processes will cause the resulting ice particles to become so large that they acquire a considerable rate of fall with respect to the water droplets, and, in their descent, they will continue to grow, not only by the evaporation-sublimation transfer of water from the surrounding water drops, but also by overtaking and coalescing with such drops as may happen to be in their path of fall. Since their size will not be limited by their rate of fall, these ice pellets can become quite large in the subfreezing layers of the cloud. When they encounter temperatures above the freezing point they will begin to melt and, if the resulting water drops are larger than the maximum raindrop size, they will break up into smaller drops—thus reaching the ground as rain.[2]

[2] If no sublimation nuclei had been present, under the circumstances assumed above, the continuance of the cooling would have resulted only in increasing the size of the cloud droplets—the cloud particles thus continuing to exist in the form of undercooled liquid drops. That this latter process cannot lead to the formation of precipitation was, however, shown by Bergeron by a series of simple calculations and considerations presented in his original paper (4).

Neither Bergeron nor Findeisen claim that the presence of subfreezing temperatures and sublimation nuclei is always necessary for the formation of precipitation. Findeison points out that if the humidities between the cloud layer and the ground are high enough, the cloud elements themselves may become sufficiently large to reach the ground as light rain or drizzle. Bergeron says that there are two other processes which may give rise to even heavy precipitation. The first process is instigated by what he calls the Reynolds effect in which those elements at the top of the cloud are cooled by radiation with a consequent reduction in the vapor pressure of the droplets so cooled and an increased condensation on them. These droplets thus acquire a size which is sufficient to cause them to fall through the lower part of the cloud and to thereby collide with the smaller and more slowly falling droplets, thus creating the observed rain. Bergeron points out, however, that in order to obtain heavy rain by this process, the cloud must have a great vertical thickness. Moreover, this process cannot set in unless some part of the cloud top is shielded from the sun's radiation.

The second explanation which Bergeron gives for the occurrence of heavy rain without subfreezing temperatures is that the electric field in the region may become so great that a coalescence of the cloud droplets is brought about by the induction of electrical charges within the droplets. In discussing the potentialities of this effect, he simultaneously considers the possibilities of the coalescence of droplets of equal size due to hydrodynamical attraction. He apparently discards hydrodynamical attraction in favor of that due to electrostatic induction on the basis of a set of computations made in "Physikalische Hydrodynamik" by V. Bjerknes, J. Bjerknes, H. Solberg, and T. Bergeron (**6**). Köhler, however, has pointed out (**7**) that the results of Bjerknes' electrostatic induction computations are too large by a factor of 10^4. It also appears that the results of his hydrodynamical computations are too small by a factor of 10^2. When these two errors are considered along with the fact that the electric field of the earth's atmosphere has been found to decrease rapidly with height above an altitude of four or five kilometers (**8**), it would seem that, assuming the remainder of the calculations to be correct, the effects of any electrostatic induction attractions which may be present must be subordinated to the hydrodynamical attraction effects in attempting to account for the formation of precipitation.

However, if validity is assumed for Schmidt's equation giving the heights of fall required for the coalescence of two equally large drops by hydrodynamical attraction (**9**), it results that this latter effect also must be of a very minor order of magnitude. In order to apply this equation, it is first to be assumed that the cloud droplets are arranged in horizontal layers and that they are all equally spaced both within the layers and with respect to the droplets in the adjacent layers in such a way that the straight lines connecting the droplets in a layer form a series of squares. This having been done, the droplets for a given layer are then assumed to coalesce as is shown in figure 1 in which: (*a*) The dots designate the initial positions of the droplets. (*b*) The crosses designate the initial positions of the droplets after the first coalescence. (*c*) The circles designate the initial positions of the droplets after the second coalescence. (*d*) The triangles designate the initial positions of the droplets after the third coalescence. (*e*) The initial positions of the droplets after the fourth coalescence.

The droplets next may be assumed to have an initial radius of 10μ—this radius being a little greater than the mean droplet radius found by Köhler in his cloud particle measurements (**10**). In order to make the most likely assumption as to the distances between the droplets, the results of the cloud particle density measurements performed by Köhler, Conrad, and Wagner (**11**) may be used. These three investigators made a total of 59 measurements of the number of cloud particles per unit volume of air—the extremes of these measurements being $20/\text{cm.}^3$ and $580/\text{cm.}^3$ and the mean value being about $64/\text{cm.}^3$ When the mean value together with the assumed initial radius is used in Schmidt's equation, it is found that it requires over 7 days for drops with a radius of 100μ to form and over 75 days are required for the formation of drops with a radius of $1{,}000\mu$. Even if the extremely great cloud particle density of $8{,}000/\text{cm.}^3$ estimated by Findeisen for cumulus clouds is assumed, it is found that over 3 hours are required for the formation of the 100μ drops and over 32 hours are necessary for the formation of the $1{,}000\mu$ drops. In view, then, of these results, and in view, especially, of the highly improbable but most favorable assumptions as to the space distribution of the drops to start with, it would seem as though coalescence of equally large drops in accordance with the ordinary laws of hydrodynamics is to be neglected as a factor contributing to the formation of precipitation.

Before discarding coalescence due to hydrodynamical attraction completely, however, the drop size distributions reported as being observed by Defant (**12**), Köhler (**13**) and Niederdorfer (**14**) are to be considered. These drop size distributions indicate that, starting with certain basic drop sizes, a series of coalescences occurs which, up to certain limits, brings it about that, in the main, the mass of the larger drops is merely that of the basic drop multiplied by some power of 2.[3] Although considerable disagreement as to the validity and accuracy of these observations exists among the observers themselves, it would seem that the very fact that the distributions have been observed by three independent investigators would warrant the acceptance of their reality. This being the case, one is then forced to conclude that the ordinary laws of hydrodynamics, upon which Schmidt's coalescence equation is founded, are not applicable for droplets of the minute sizes composing these distributions. This being agreed upon, the question now remains as to whether or not, drops of the maximum size observed in these distributions having been produced, the larger drops of rain can be formed by coalescence in accordance with Schmidt's equation—it being assumed that Schmidt's equation is valid for the drops whose sizes are greater than those within the size-distribution range. Consulting the results of the observations of Niederdorfer (who has conducted the most recent and, to all appearances, the most reliable set of size distribution observations) it is found that the size distribution no longer appears for drops whose radii are greater than, say, 640 μ. It is hence to be determined whether drops with radii equal to or greater than 1,000 μ can be formed by coalescence in accordance with Schmidt's formula—the 1,000 μ radius being chosen since Niederdorfer found that almost 20 percent of the drop sizes measured during showers and thunderstorms exceeded this limit. In making this calculation it seems justifiable to assume that the spacing will be the same as that assumed in the preceding application of Schmidt's equation—allowing, of course, for the increased spacing

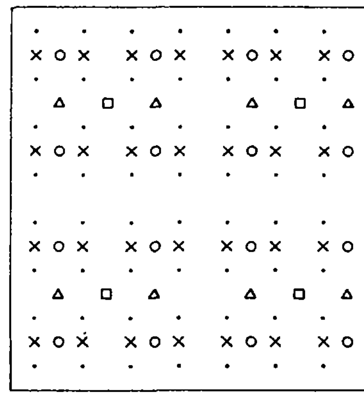


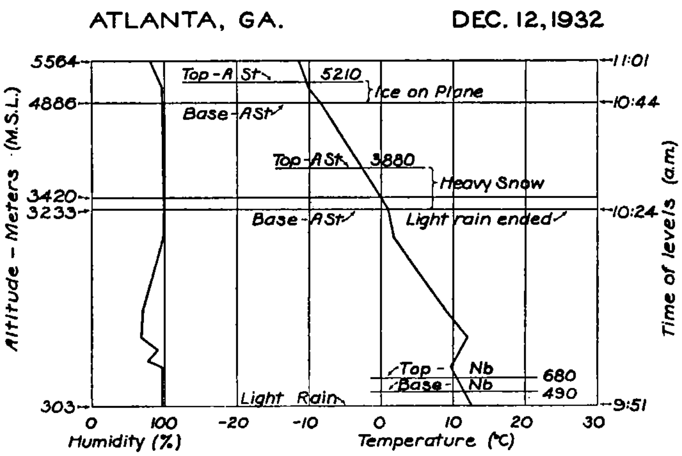
Figure 1

between the drops as a result of the coalescence occurring within the size-distribution range. On the basis of this assumption—all other assumptions being the same as for the first application of Schmidt's equation—it is found that with the average drop spacing for the observations of Köhler, Conrad and Wagner, about 5 weeks are required for the formation of the 1,000 μ drops, while with the minimum drop spacing estimated by Findeisen for thunderstorm clouds, 15 hours are necessary to produce the 1,000 μ drops from the 640 μ drops. It therefore appears that coalescence due to hydrodynamical attraction cannot produce the larger drops even when coalescences within the drop size distribution range are conceded to take place in another manner than that prescribed by the ordinary laws of hydrodynamics.

In support of the main feature of the Bergeron-Findeisen theory it is to be said that, if, as is usual, it is admitted that the condensation nuclei of the earth's atmosphere consist of minute droplets of salt or acid solution, it can be definitely asserted that, in some cases at least, the sublimation nuclei are quite distinct from the nuclei on which condensation takes place. The foundation for this assertion lies in the fact that, according to Wegener (**16**), the water obtained by melting snow taken from the firn region of a glacier does not conduct electricity. That sublimation nuclei must, in general, have a nature which is different from that of condensation nuclei, is indicated by the following considerations which are due, in the main, to Wegener (**17**), (**18**): In the first place, the molecular structure of solids and crystals is considerably more complicated than that of the liquids. This means, of course, that the

[3] According to Köhler, such a distribution occurs for four basic drop sizes—the masses of the basic drop being related as 2, 3, 5, and 7, respectively (15).

collisions of the molecules which are favorable enough to produce a crystal are much more improbable than those which would produce a liquid drop. Secondly, considering the formation of a solid from an under-cooled liquid, it is observed that, although the introduction of a solid body usually serves to bring about such a formation, not all solid bodies have the same ability in this respect, and that the more carefully the body is rounded off and smoothed, the less capable it is of bringing about a "release" of the undercooling. Evidence as to the truth of this assertion is furnished by the fact that water can be undercooled in a smooth-walled glass vessel and that substances having sharp edges and being isomorphous with the crystalline form of the undercooled liquid possess the best releasing capabilities. Since, then, the nature of the resulting solid is the same regardless of whether it is formed by freezing from the undercooled state or by sublimation from the gaseous state, it would then seem that the effectiveness of the sublimation nuclei must be governed by the same laws



Light rain at surface at 9:44 a.m., diminishing to sprinkling at 10:50 a.m., and continuing at 11:35 a.m.
Figure 2

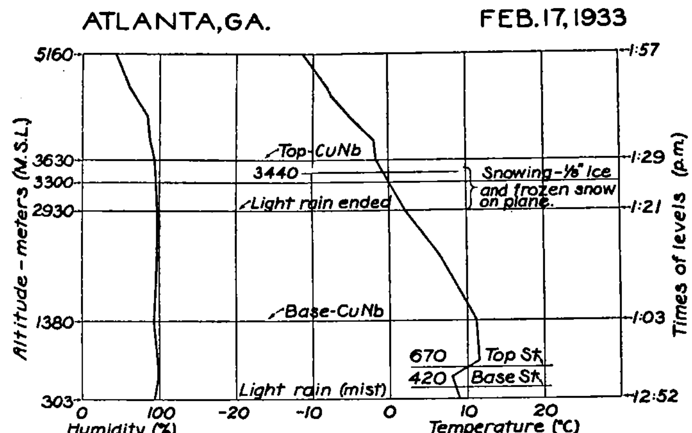
as the "releasing effectiveness" of foreign bodies in the case of undercooled liquids.

Indirect evidence as to the prevalence of the Bergeron-Findeisen process in the formation of precipitation may be obtained in two ways. The first of these is the correlation of the salt and acid content of rain with the intensity of the rainfall, i. e., if it is assumed, with Findeisen, that ice particles cannot be formed in the atmosphere by the spontaneous freezing of undercooled drops.[4] If, as is supposed by Bergeron and Findeisen, most of the heavy rain originates as ice particles, a low salt and acid content would be expected with high rainfall intensities while the rain collected from light intensity falls of rain would be more likely to have a high acid and salt content. Unfortunately, however, there have been no simultaneous determinations of the salt and acid content which can be correlated with the intensity of the rainfall. However, in his paper on the chlorine content of rain, Israel (**20**) published the following set of chlorine determinations with the corresponding rainfall intensities in order to show how the chlorine content may vary within a single fall of rain:

TABLE 1.—*Strong upglide rain Leyden, Holland—Sept. 23, 1932*

Time (a. m.)	Amount (inches)	Mg. Cl/liter
6:00 to 9:15	0.44	0.8
9:15 to 9:30	.17	.47
9:30 to 9:45	.05	1.57
9:45 to 10:00	.03	1.69
10:00 to 10:15	.03	1.58

As is indicated in the table, the collection of the water for the first analysis terminated at 9:15 a. m. After this, the water for the various analyses was collected at 15-minute intervals. It will be seen that, considering only the period throughout which the water was collected at 15-minute intervals, a well-defined inverse relationship exists between the amount of rain in the interval and the corresponding chlorine content. The high chlorine content found for the rain caught from 6 a. m. to 9:15 a. m. may well be explained in either or both of two ways. First, the average amount of rain for 15-minute intervals during this period is only 0.03 inches, which, on the inverse relationship hypothesis, would call for a high chlorine content. Secondly, making the likely supposition that the actual rainfall intensities varied widely from the mean during this period, this high chlorine content could also have resulted from the cleansing of the impurities from the air by the first part of the rainfall. If this is the accepted explanation, it is to be noted that, assuming no marked change in the direction and speed of the wind, this possibility cannot be used to explain the high chlorine content of the last three of the 15-minute intervals, since the air has presumably already been washed by the preceding part of the rainfall. It therefore appears that the high chlorinity for the last 45 minutes of the rainfall is only to be explained on the basis of the inverse relationship concept—which is in accordance with the Bergeron-Findeisen theory.[5]



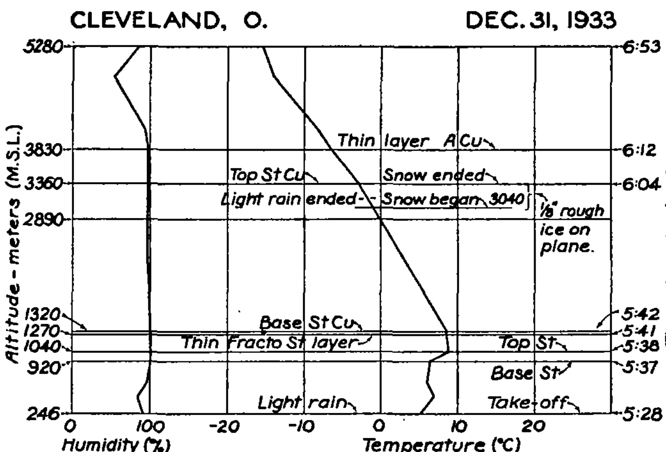
Light mist at 12:45 p.m., changing to light rain at 1:55 p.m., and to sprinkling at 2:29 p.m.
Figure 3

The second test as to the prevalence of the Bergeron-Findeisen process in the formation of precipitation is that

[4] It may be contended that this assumption is incompatible with the findings of Dorsey (19) as to the existence of a spontaneous freezing point for every sample of water. It is to be pointed out, however, that, according to the account of his experiments, the samples tested were not shielded from the mechanical disturbances which might have been caused by the action of microseisms and that although it was found that certain types of mechanical disturbances were without influence on the temperature of the freezing point, other types were found to be extremely effective and that it therefore appears possible that the spontaneous freezing observed by Dorsey could have been induced under the influence of the microseisms. Since the cloud droplets are, of course, shielded from any such influence, Dorsey's finding of a spontaneous freezing point for his water sample does not, it would seem, indicate that such a spontaneous freezing point also exists for cloud droplets.

[5] It is to be remarked that even on the basis of the Bergeron-Findeisen theory, it is to be expected that the resultant rain will contain some chlorine—this being true since the Bergeron-Findeisen process involves the coalescence of the descending ice particles or melted ice particles with the drops in the lower part of the cloud. Besides this, as has been pointed out, the descending drops will acquire an additional amount of chlorine due to the impurities in the lower atmosphere.

of examining the records of the aerological airplane ascents made when rain was occurring to determine whether or not the clouds from which the rain was falling had their upper limits above the zero degree centigrade isotherm. That the presence of the zero degree centigrade isotherm within the cloud layer is sufficient, in some cases, at least, to satisfy the hypothesis of the Bergeron-Findeisen theory is indicated by the consideration of the aerograms shown in figures 2, 3, 4, and 5. The only questionable region in the interval of subfreezing temperatures is, of course, that immediately below the freezing point. That sublimation can take place on the sublimation nuclei at these



Light rain at surface throughout flight.

Figure 4

comparatively high temperatures is shown in the following way: In figures 2, 3, and 4 it will be seen that snow was forming in clouds which had temperatures higher than −3° C. at the top. Now, according to the theory as developed by Wegener (**21**) [which theory has, in the main, been confirmed by the recent experiments of Nakaya of Japan (**22**)], the formation of snow requires a more intense supersaturation with respect to ice than the formation of plain ice crystals (the German *vollekristalle*). Since, according to these observations, it was possible to obtain these higher supersaturations within the temperature interval from zero to −3° C., without having the excess water vapor absorbed by condensation on the cloud droplets, it therefore seems that the smaller supersaturations necessary for the formation of plain crystals without having supersaturation with respect to any liquid droplets that may be present. The truth of this last assumption is well demonstrated in considering the observation shown in figure 5. Here, it will be seen that what the pilot describes as a "few small pellets" of ice were observed at the top of a cloud whose indicated temperature was as high as −0.2° C.—thus apparently demonstrating the validity of the assumption that sublimation can take place at temperatures very near to that of the freezing point.[6]

In selecting the stations for this examination, all of the southern stations which rendered a report as to the surface conditions at the time of the flight and which had a latitude of less than 35° were chosen. Besides these, certain northern stations which were reputed to have made a large number of bad weather flights were also selected. The results of this investigation are shown in the following table:

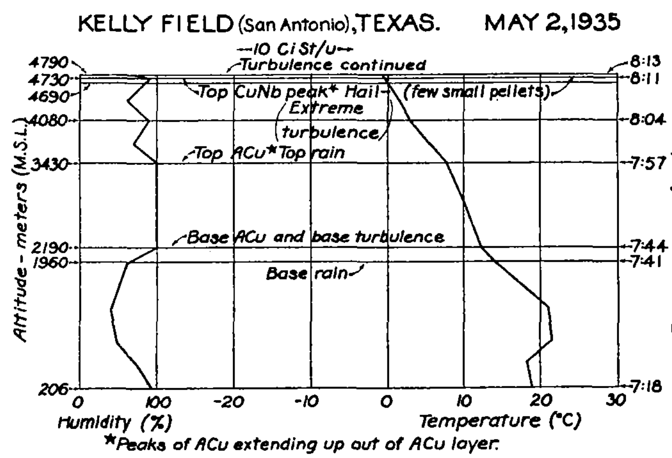
TABLE 2

Number of cases in which—	Summer (April-September, inclusive)		Winter (October-March, inclusive)		Total
	Southern stations	Northern stations	Southern stations	Northern stations	
1. Precipitation was actually observed at a higher altitude than the 0° isotherm	61	35	79	29	204
2. Clouds from which precipitation presumably was falling were observed above the 0° isotherm	25	20	18	25	88
3. Light rain or drizzle was falling from low clouds containing no subfreezing strata	3	2	5	0	10
4. The theory is neither supported nor contradicted due to the altitude of the cloud top and the upper limit of the precipitation being unknown	12	11	8	5	36
5. One or both cloud limits and precipitation limits coincide (and which, therefore, are assumed to be cases of "wet" clouds)	4	1	7	0	12
6. Special considerations are required	6	0	4	0	10
Total	111	69	121	59	360
Total number of effective observations	99	58	113	54	324

Southern stations: Atlanta, Dallas, El Paso, Galveston, Miami, Montgomery, San Antonio, and Shreveport.
Northern stations: Billings, Chicago, Cleveland, Pembina, Sault Ste. Marie.

In this table, the term "number of cases" refers to the number of airplane observations for which the observation of rain or drizzle was reported by the pilot during the flight or by the observer on the ground—all records up to and including the year of 1937 being used.

If, now, the cases classified in the fourth of the six categories are discarded, we may call the remaining number of observations the number of "effective observations."



*Peaks of ACu extending up out of ACu layer.

Figure 5

It will then be seen that of these 324 effective observations, 302 are not contradictory to the requirements of the Bergeron-Findeisen theory. Furthermore, on the basis of the assumption made in connection with the fifth category, the 12 cases listed under it may be regarded as not being contradictory to the Bergeron-Findeisen theory.[7]

[6] The conclusion reached in this paragraph, of course, assumes—again with Findeisen—that spontaneous freezing is nonexistent in the atmosphere. *If, as is believed by many physicists, some mechanical disturbance is required to produce the freezing of subcooled water,* it is quite possible that some of the ice pellets may have been formed due to the collision of subcooled drops. It does not, however, seem to be probable that this process could lead to the formation of a noticeable number of such pellets.

[7] The term "wet cloud" used in describing the clouds encountered in the flights of this category means, of course, that these clouds contained drops which were large enough to penetrate the boundary layer of air adjacent to the windshield, say, of the plane but which at the same time were not large enough to fall through the layer of dry air between the cloud and the ground without evaporating. It appears allowable to assume that the sizes of these drops lay within or not far from the "size distribution range" of drop coalescence previously discussed and that, therefore, they could have been formed by the type of hydrodynamical-attraction coalescence mentioned there.

The permissibility of this latter assumption is well demonstrated by the report of the pilot for the flight whose results are shown in figure 6. In this case, as will be seen, the pilot reported entering a stratus overcast at 100 meters above the ground, and then, while still in this stratus he reported striking heavy rain at 375 meters above the ground—both the rain and the stratus being reported as ending at 620 meters above the ground. A consultation of all available records reveals that no rain fell during the period of the flight—thus indicating that a pilot may even go so far as to term a wet layer of the cloud "heavy rain." This, then, leaves the 10 cases of the sixth category to be accounted for.

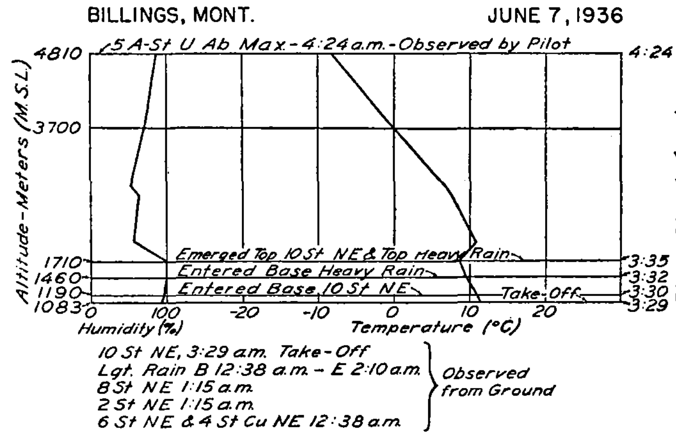


Figure 6

In four of these cases, the temperatures indicated at the top of the cloud layer were 1° C. or less above the freezing point. Since the error in the calibration of the temperature elements may be as much as 2° C., it is therefore possible that, for these four cases, the required subfreezing temperatures could have been present.

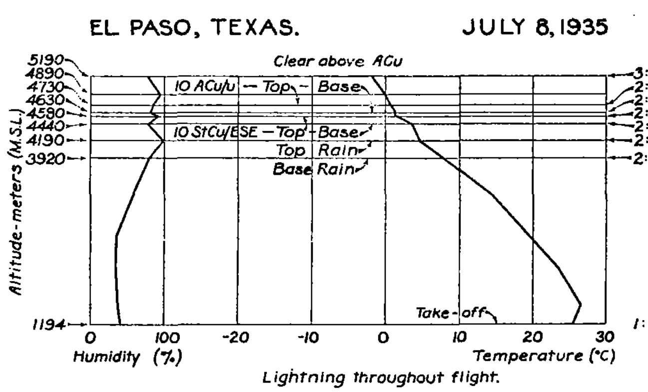


Figure 7

Two more of the cases in the sixth category are shown in figures 7 and 8. In these two cases, an increase in the humidity and fairly good lapse rates make it appear that, considering the tolerances for instrumental error just mentioned, the upper cloud limit really could have been above the 0° C. isotherm although the pilot's reports indicate the upper cloud limit to be below this isotherm. Bearing in mind the multiplicity of the duties of the weather flight-pilots, and bearing in mind also the trying conditions under which these bad-weather flights were made, it is to be expected that, in the 360 cases investigated, some of the pilot's reports will be in error. That there should be two cases of this nature is therefore not surprising.

Figures 9, 10, 11, and 12 show the remaining four of the 10 cases. In the flight of figure 9 the pilot merely states that clouds were encountered at about 2,000 feet and that rain was encountered at about 10,000 feet without indicating whether he left the lower cloud layer or the rain and, if so, when. Considering the scarcity of the notes along with the probability of their inaccuracy—as is revealed, for instance, by the lack of saturation at the stated elevation of the cloud base—no definite conclusions appear to be warranted, and it would seem that this flight could,

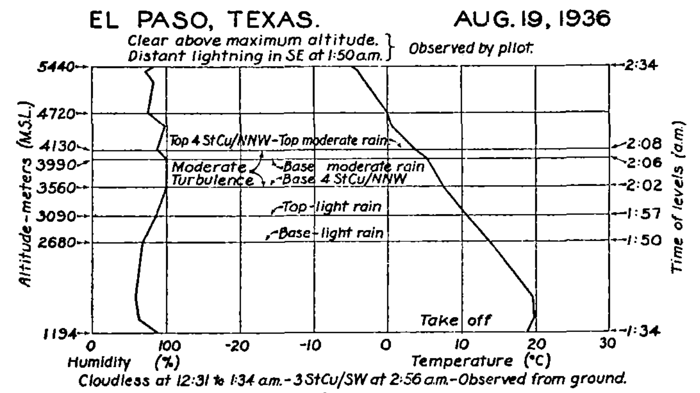


Figure 8

therefore, be classified with those flights which neither confirm nor deny the theory being evaluated.

The difficulty with the flights shown in figures 10, 11, and 12 is, of course, that rain—light though it is—is observed at the surface even though the zero degree isotherm is above the cloud layer from which the rain appears to be coming and even though low humidities exist between the base of the cloud layer and the ground. In all three cases, the thickness of the cloud layer would seem to be great enough to account for the formation of the rain either by the Reynolds effect or perhaps by coalescence within the size-distribution range. Although the flight

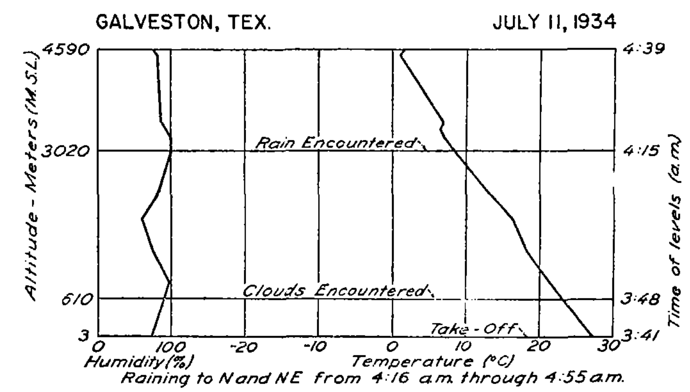


Figure 9

shown in figure 12 was made in daylight, attributing the formation of the rain to the Reynolds effect is not excluded here since the pilot's report shows that there were scattered tops of the "stratus" extending considerably above the general layer of the "stratus"—which means that those portions of the top of the general layer which were in the shade of these scattered tops might have been losing a sufficiently great amount of heat by radiation for the Reynolds effect to set in and produce the occasional light rain at the surface. However, it will be noted that in both figures 11 and 12, no inversion exists at the top of the cloud layers. If the Reynolds effect were active, one might reasonably expect that its activity would be evidenced by the presence of such an inversion. But if certain fairly plausible assumptions are made, it can be shown that this is not necessarily the case. The required assumptions are, briefly, that, first, in accordance with the results of the water content measurements of Köhler, Conrad, and Wagner (**11**), the mass of the liquid water and the mass of the water vapor in a cloud are of the same order of magnitude; and second, that, in accordance with an assertion made by Brunt (**23**), no great change is produced in the emissive power or absorptivity of liquid water

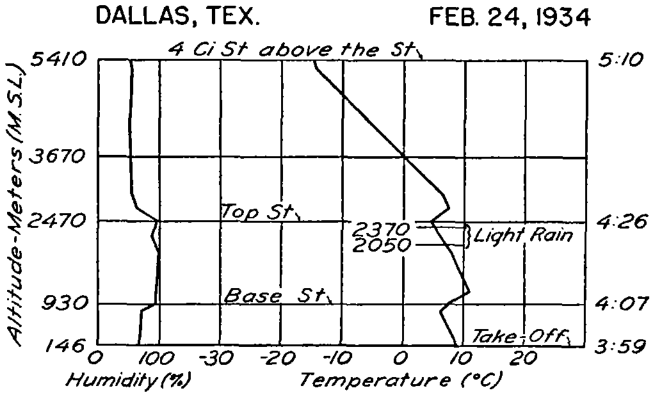
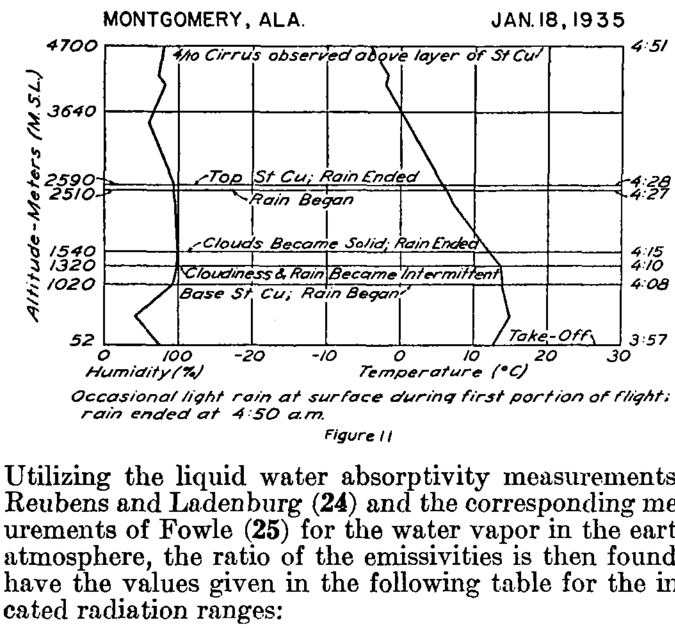


Figure 10

by the fact that it consists of small drops such as those found in fogs and clouds. These assumptions having been made, an application of Kirchhoff's law shows that the emissive power of the liquid water drops has the same ratio to the emissive power of the water vapor as the absorptivities of liquid water and water vapor, respectively.



Occasional light rain at surface during first portion of flight; rain ended at 4:50 a.m.

Figure 11

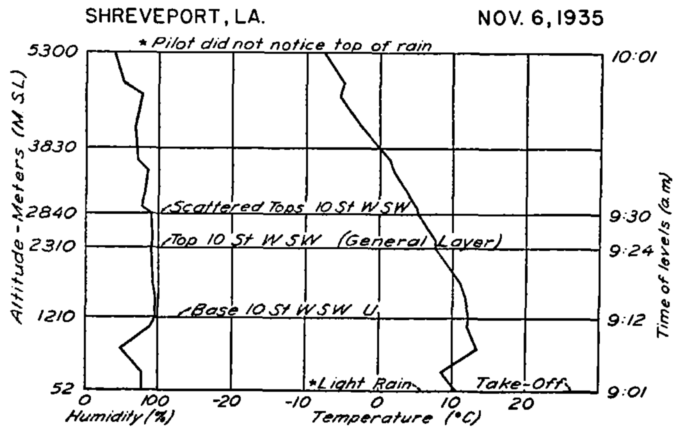
Utilizing the liquid water absorptivity measurements of Reubens and Ladenburg (**24**) and the corresponding measurements of Fowle (**25**) for the water vapor in the earth's atmosphere, the ratio of the emissivities is then found to have the values given in the following table for the indicated radiation ranges:

Ratio of emissivity of liquid water E_l to emissivity of water vapor E_v

Wave length (microns)	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18
E_l/E_v (0.001 cm. precipitable waters)	15.6	4.9	10.2	5.0	10.2	∞	∞	∞	∞	∞	∞	---	---	---	---
E_lE_v (0.06 cm. precipitable water)	2.8	2.5	2.0	1.1	1.9	25.0	∞	∞	100.0	100.0	16.7	4.0	2.2	2.0	1.3

Considering the ratios given for the smaller quantities of liquid water and water vapor (which, of course, are those most nearly applicable to the conditions in question), it will be seen that this ratio is quite large for all the radiation ranges. This, then, means that the cloud droplets can cool more rapidly by radiation than the surrounding air and that, as a consequence, it seems possible that the water droplets themselves may experience a loss of heat by radiation without the occurrence of a corresponding loss of heat in the air surrounding the droplets. When it is additionally borne in mind that, under the assumed conditions, a minute fall in the temperature of the droplets will result in a corresponding condensation of the vapor surrounding the drops on the drops together with a corresponding liberation of the heat of condensation, it would consequently seem that the action of the Reynolds effect is not necessarily accompanied by the formation of an inversion.

It will finally be noted that for at least one of these three cases (that shown in fig. 11) rain is reported as being encountered very near the top of the cloud layer. On first consideration, this phenomenon also does not appear to be explainable by any of the processes which



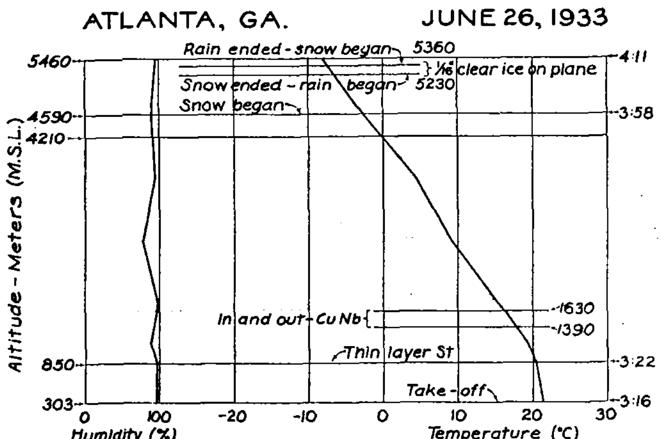
Occasional Light rain to 9:10 a.m.
R B 12:30 a.m. R Cont'd.

Figure 12

have been listed thus far. For, both in the case of coalescence within the size distribution range and in the case of the action of the Reynolds effect, a considerable fall of the coalescing droplets with respect to the surrounding air—and therefore with respect to the unused nulcei—is required before drops large enough to be accounted as rain result, and, since there is no reason to suppose that condensation will not continue to take place on the portion of the unused nuclei which are thus ascending with respect to the coalescing droplets, it would therefore seem that none of the processes so far outlined serves to explain this phenomenon. If, therefore, the phenomenon is real, the existence of some unknown rain formation process would seem to be indicated. However, if the circumstances under which these flights are made are borne in mind, it would seem that there is a considerable chance that the phenomenon may no be real. For, in the first place, due to the large horizontal component of the velocity of the plane with respect to the surrounding air, the observed variations in the weather may frequently be those with respect to the horizontal rather than with respect to the vertical. In the second place, owing to the multifarious duties of a pilot in these bad-weather flights, it is quite conceivable that changes in the weather (and gradual changes in particular) may set in considerably earlier than the time at which they are observed by the pilot—this being especially the case if the attention of the pilot is not confined to the occurrence or nonoccurrence of the phenomenon in question. It is therefore quite possible that, in the case being considered, the pilot may have flown under the crest of one of the rolls of the strato-cumuli (at the top of which the action of the Reynolds effect would, of course, be considerably more intense than it would in those portions of the upper cloud surface which intervene between these crests) at the time at which the beginning of the rain was observed and that he also emerged from the strato-cumulus layer in one of the troughs in between these crests therewith failing to notice the gradual diminution of the rain owing to his absorption in the remainder of his duties connected with bad-weather flying.

The only way to be sure in instances of this sort is, of course, to devise a means of measuring drop sizes in connection with these flights. Such a procedure does not appear to be impossible.

Besides the foregoing indirect evidence as to the prevalence of the Bergeron-Findeisen process in the formation of precipitation, a consideration of the flights shown in figures 2 and 13 furnishes evidence as to the existence of this process which is somewhat more direct. In figure 2 it will be noted that an accumulation of ice was obtained



Moderate to heavy rain at surface from 3:38 a.m. to 4:15 a.m.; light rain from 4:15 a.m. to 4:48 a.m.

Figure 13

in a layer of alto-stratus which lay considerably above the cloud layer from which snow was falling. Since the presence of liquid drops is necessary for the formation of ice on aircraft, we thus have a case of the existence of liquid drops at a temperature lower than that at which snow was forming. As far as the author is aware, the only explanation for this is that effective sublimation nuclei were lacking at the higher levels and hence under-cooled droplets instead of ice crystals or snow flakes were formed. In figure 13, it will be seen that the pilot in his ascent first encountered snow and then rain and finally snow again just before he reached the top of the flight. Again, such an alternation in the occurrence of water in the solid and liquid states can, it would seem, only be accounted for by the lack of effective sublimation nuclei in the region in which the liquid drops were formed.[8] These two cases, therefore, furnish fairly positive evidence as to the occurrence of the Bergeron-Findeisen process and it thus follows that considerably more importance than otherwise may be attached to the circumstantial evidence furnished by both the chlorine content observations and by the data as to the relative altitudes of the tops of the precipitation producing clouds and those of the 0° C. isotherm.

In closing, a discussion of this nature would not be complete without a consideration of a criticism of Bergeron's theory published by Holzman in 1936. (26) Those portions of the criticism which deal with the theoretical aspects of Bergeron's theory, have, in general, been answered by the developments in the theory, subsequent to the publication of Holzman's article. A closer examination of the two examples[9] which he cites as being contrary to the theory will, however, be found to be worth while. As the first of these examples, he gives the following:

> On June 15, 1936, in a flight made from Albany to Newark during the hours 5 to 6 a. m., a moderate rain was encountered in ascending and descending through a strato-cumulus deck. There were some low ragged stratus clouds extending from 600 to approximately 1,500 feet with the base of the strato-cumulus near 1,800 to 2,000 feet but frequently merging with the low stratus. The flight was made at 8,000 feet with the temperature at or near 45° F. At this elevation the plane was generally above the cloud deck but, due to the undulating upper surface, an occasional cloud roll would submerge the ship. Aloft were a few cirrus and a few altostratus clouds that thickened to a near overcast far to the east, but precluded the possibility that the rain that was encountered both on ascent and descent could have originated from an upper cloud system. Upon approaching Newark the strato-cumulus layer seemed to be rapidly dissipating, and by the time the landing was made the sky condition could be described as broken.

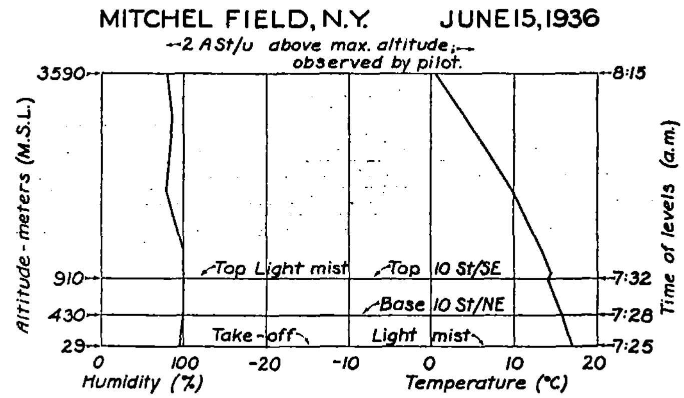


Figure 14

> The 8 a. m. synoptic chart indicated 0.08 inches of rain at Albany and 0.32 inches at New York City. The Mitchell Field sounding on June 15 taken at 7 a. m. reached a height slightly over 11,500 feet at which elevation the temperature was 34° F. Extrapolation of the lapse rate curve would place the freezing isotherm well above 12,000 feet. The temperature at 8,000 feet was 46° F., in very good agreement with the temperatures as observed during the above-mentioned flight at this altitude. The cloud observations indicated only two-tenths altostratus above a rather low overcast stratus deck that extended from 1,500 to 3,000 feet.

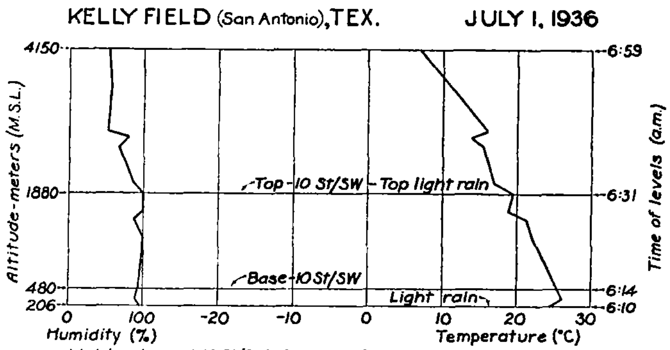
Regarding this flight it is to be considered that moderate rain was not reported either at Albany or New York at the times in question. The 0.08 inch of rain mentioned at Albany occurred between 1:00 and 5:00 p. m. of the 14th—only a trace being recorded from 6:08 a. m. to 8:56 a. m. of the 15th. Also, the bulk of the 0.32 inch of rain reported at New York City occurred before the night observation of the day before. Only 0.06 inch occurred after this, and all of this occurred before 2:30 a. m. of the 15th—traces of rain being reported from then until 8:45 a. m. Furthermore, the Mitchel Field aerograph flight shown in figure 14 only indicates "light mist" between the cloud layer and the ground—the humidity throughout the stratum being approximately 100 percent.

It would therefore seem that the "moderate rain" encountered in the strato-cumulus during this flight was probably a very light rain due to one of the two processes

[8] It is to be noted here that this alternation of rain and snow was apparently one with respect to the horizontal instead of with respect to the vertical, and that, furthermore, the observed rain could not have been formed by the Bergeron-Findeisen process since this process requires a melting of the snow flakes or ice crystals and, owing to the altitudes and temperatures at which it was observed such a melting is quite improbable.

[9] These are the examples referred to by "C. F. B." in the Bulletin of the American Meteorological Society (27) where, in his account of the proceedings of the 1939 meeting of the Institute of the Aeronautical Sciences (at which the main part of the above considerations was presented in connection with their application to the aircraft icing problem), he says that: "H. G. Houghton and Ben Holzman, however, pointed to the occurrence of rains from clouds entirely above freezing, which does not permit so simple an explanation of precipitation."

already mentioned as being alternate to the Bergeron-Findeisen process and that, as in the case of the Billings flight previously mentioned, the apparent intensity of the rain was increased by the speed of the plane. Judging by the Mitchel Field ascent, this case would therefore be listed in that category of table 2 which was allotted to those cases in which light rain or drizzle was falling from low clouds with high humidities between the earth and the cloud.



Light rain and 10 St/SW observed from surface at: 5:12 a.m., 6:10 a.m., 7:08 a.m. Rain ended -7:52 a.m.

Figure 15

The second example mentioned by Holzman is shown in figure 15. As is indicated, light rain was reported both by the observer on the ground and by the pilot, and the humidities between the cloud base and the ground lay between 92 percent and 97 percent. The San Antonio precipitation record for the early part of the day of the flight reads as follows:

Period:	*Amount of rain*
Midnight–1 a. m.	0.02 inch.
6 a. m.–7 a. m.	trace.
7 a. m.–8 a. m.	0.01 inch.

In compiling table 2, therefore, this case also came under the third category, i. e., in the category of being, therefore, compatible with the theory as outlined by Findeisen.

Summarizing then it has first been shown that, assuming Schmidt's equation for the distance of fall required for the coalescence of two equally large drops by hydrodynamical attraction to be valid, the process which has been the main rival of the Bergeron-Findiesen process, i. e., the coalescence of drops of equal size—cannot produce the large drops which are observed in heavy rains—this being true even if, in consideration of the drop size measurements of Defant, Köhler, and Niederdorfer, such a coalescence is conceded to have previously taken place up to the top of the range in which the size distributions indicative of such a coalescence are observed. Secondly, it has been pointed out that the nonconductivity of the water obtained by melting the snow taken from the firn region of a glacier indicates that, in some cases at least, the duality of the nuclei required for condensation and sublimation is real, and it has been further pointed out that such a duality is to be expected from a consideration of the more complicated molecular structure of solids as compared with liquids. In the third place, it has been shown that such indirect evidence as is available, i. e., that to be derived from the chlorine content observations and that derived from the data as to the relative altitudes of the top of the precipitation producing clouds and those of the zero degree centigrade isotherm—points to the prevalence of the Bergeron-Findeisen process in the production of rains of any considerable intensity. Fourthly, it has been indicated that the only apparent explanation for the appearance of undercooled water drops at higher and colder altitudes than those at which snow is simultaneously observed is that effective sublimation nuclei are lacking in those parts of the atmosphere in which the undercooled drops originate—this phenomenon also, therefore, confirming the existence of the Bergeron-Findeisen process in the earth's atmosphere and lending considerably greater weight to the circumstantial evidence previously presented. Finally, it has been demonstrated that a more detailed consideration of the examples cited by Holzman as being contrary to the theory shows that such is not the case at all.

CONCLUSIONS

On the basis of the evidence presented, it therefore must be concluded that the Bergeron-Findeisen process actually takes place in the atmosphere. Furthermore, the results of the chlorine content observations together with the relationship of the altitudes of the 0° isotherm to the altitudes of the tops of the precipitation-producing clouds seem circumstantially, to indicate that the process is, at least, the main one in the production of rains of any considerable intensity and that any alternative processes, such as the action of the Reynolds effect and coalescence within the size-distribution range, are confined mainly to the production of light rains and drizzles. As has been suggested, however, the inferences drawn need to be confirmed by more accurate observations—it being particularly necessary to judge the occurrence or nonoccurrence of rain as observed from an airplane by some other means than by the amount of water striking the plane. Also, of course, an investigation as to the nature of the sublimation nuclei is needed. When this has been done, it would seen as though it should be possible ultimately to considerably extend the accuracy of precipitation forecasts.

ACKNOWLEDGMENTS

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TROPICAL DISTURBANCES OF OCTOBER 1940

By JEAN H. GALLENNE

[Weather Bureau, Washington, November 1940]

October 20–23.—The earliest indications of this disturbance were contained in an observation from the S. S. *Cristobal* during the evening of October 20. The vessel, which was a short distance north of the Canal Zone at that time, reported that she experienced cloudy weather with southwest wind, force 5 (Beaufort Scale) and a barometer reading of 1,008 millibars (29.77 inches).

The depression progressed in a northwesterly direction and was centered near latitude 11°30′ N., longitude 79°-30′ W., on the morning of the 21st. Later that day reports of high winds and gales, accompanied by moderate to heavy rains, were received from several vessels in the central Caribbean. The Honduran S. S. *Contessa* reported a barometer reading of 995.3 millibars (29.39 inches) and northeast gales, force 9, with very rough seas, near latitude 12°35′ N., longitude 80°25′ W., during the afternoon of October 21. The lowest barometer, 982.7 millibars (29.02 inches) was read on the Hawaiian S. S. *Contessa* during the morning of the 22d in lat. 12°50′ N., longitude 81°45′ W.

The disturbance continued to move in a northwesterly direction during the next 36 hours, attended by fresh to strong gales.

At 7:30 a. m. of October 23, the center of the disturbance was located near 14°15′ N., 82°45′ W., from which point it curved to the west and southwest, passing inland a short distance to the south of Puerto Cabezas. A report received by the Standard Fruit Co. indicates that considerable damage occurred on the northern coast of Nicaragua.

October 24–26.—On the morning charts of October 24, an area of low barometric pressure was general in the vicinity of the Greater Antilles. Subsequent ships' reports of that day indicated that a slight disturbance, 1,008 millibars (29.77 inches), with definite cyclonic wind circulation, had formed southeast of Inagua. The depression moved toward the north and north-northeast for a period of about 12 hours, then recurved sharply to the northeast and was centered near latitude 25° N., longitude 70°30′ W., on the morning of the 25th. During the following day it moved very rapidly over the extratropical waters of the North Atlantic Ocean, where, due to a lack of vessel reports, its identity was lost near 35° N., 55° W.

From reports at hand, indications are that no unusually low barometer readings were noted.

No reports of loss of life were received in connection with these disturbances, and it is very doubtful if either developed to hurricane strength.

Timely warnings and advisories were issued by the forecast center at Jacksonville, Fla., covering the movements of both disturbances.

A chart showing their tracks is herewith.

Tracks of tropical storms of October 1940.